A High Performance High Order Curvilinear Finite Element Framework for Coupled Electromechanics

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(Doctoral Candidate: 2013 - Present)

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Objectives

- ⋆ To develop monolithic and staggered approaches for coupled electromechanical systems
- ⋆ To utilise high order accurate finite elements
- ⋆ To develop mesh deformation techniques for accurate represenatation of CAD boundary for curvilinear FEM
- ⋆ To implement massively thread, data and instruction level parallel algorithms for heterogeneous architectures
- ⋆ To facilitate automatic finite element computations through a domain-aware tensor contraction framework

Robust Curvilinear Mesh Generation

PostMesh (Open Source) | C++, Cython & Python APIs | 149,000 Unit-test Cases

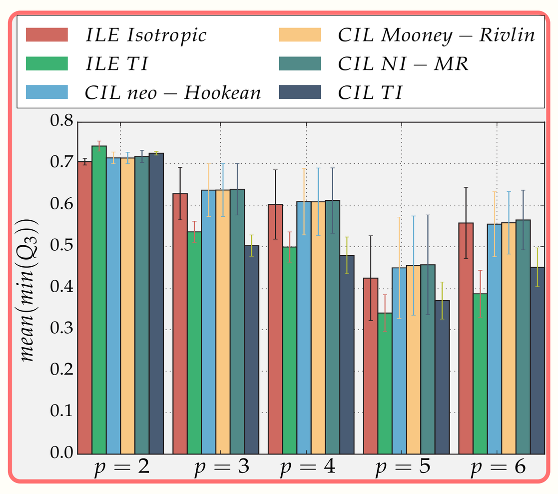
- ⋆ A multi-level approach to curvilinear mesh generation
- ⋆ A set of fundamental mesh quality measures are introduced
- ⋆ Capable of producing isotropic, anisotropic and boundary layer meshes

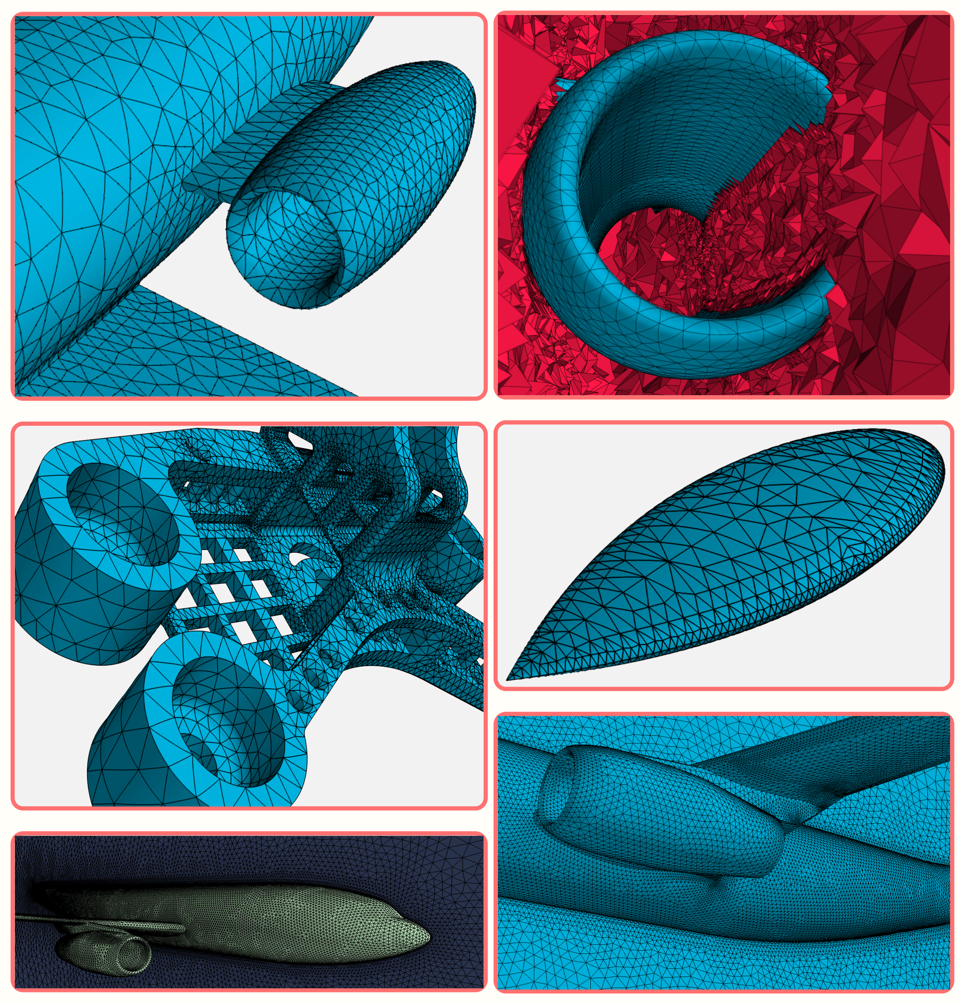
$d\boldsymbol{x} = \boldsymbol{F}d\boldsymbol{X}$ → Edge distortion

$d\boldsymbol{a} = \boldsymbol{H}d\boldsymbol{A}$ → Surface distortion

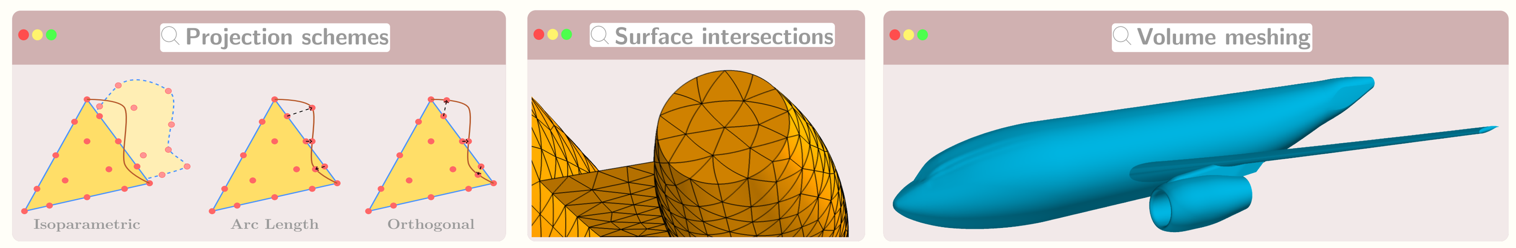
$dv = J dV$ → Volume distortion

- ⋆ A suite of solid mechanics analogies



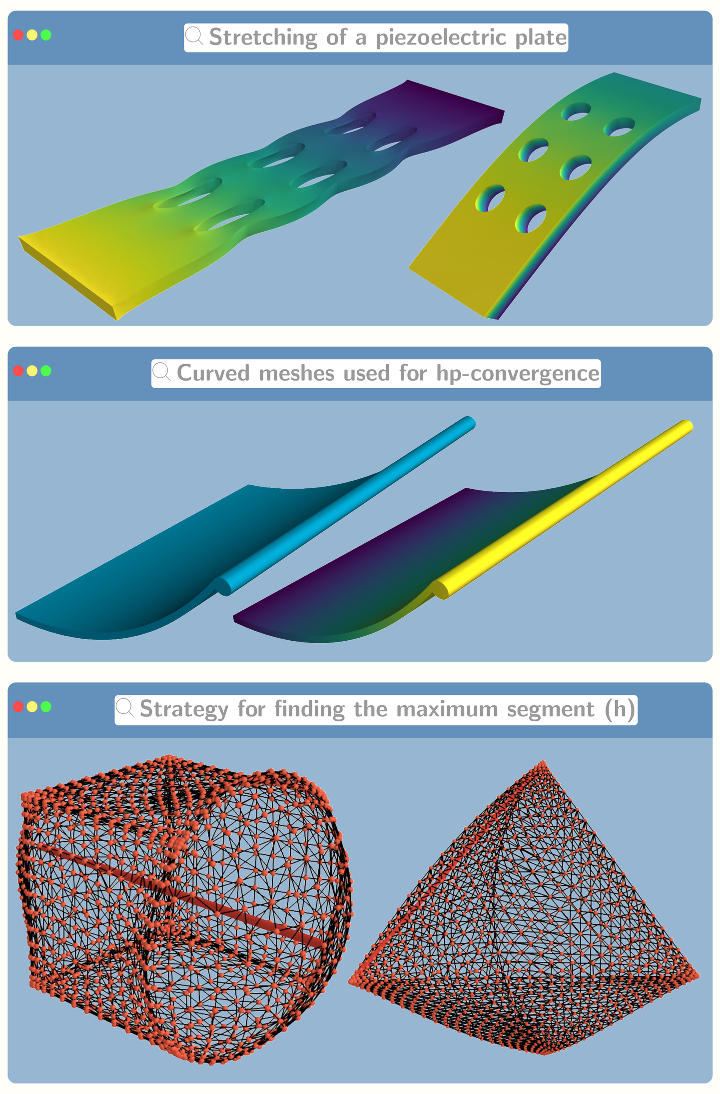


- ⋆ Various node distribution schemes



Staggered Appoach for Small Deformation Electromechanics

FEAPB (Open Source) | C++11, GUI



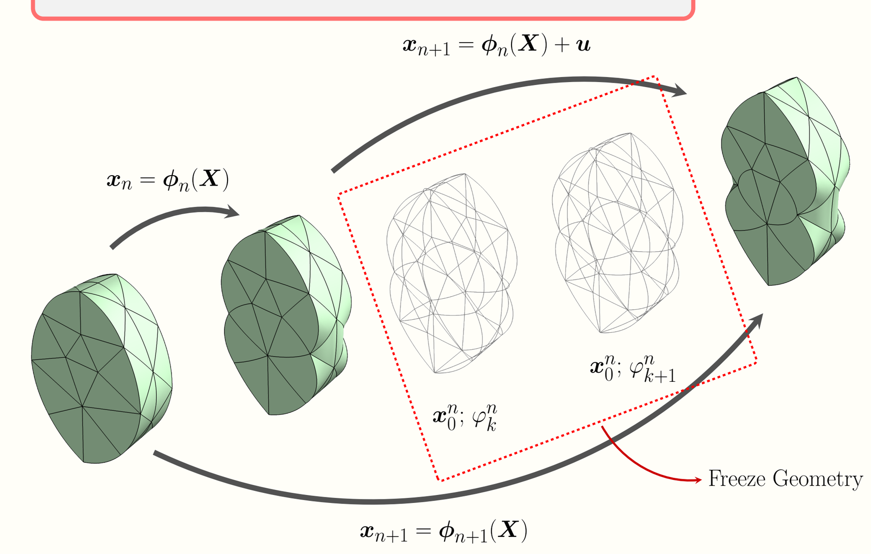
Efficiency compared to monolithic approach

Deformation	R. Error	R. Speed-up	N of Increments
Small ($< 1\%$)	$1e-06$	12.673	2
Medium ($> 30\%$)	0.0005	4.548	18
Large ($> 70\%$)	0.100	0.8137	204
Giant ($> 200\%$)	0.500	0.0452	1273

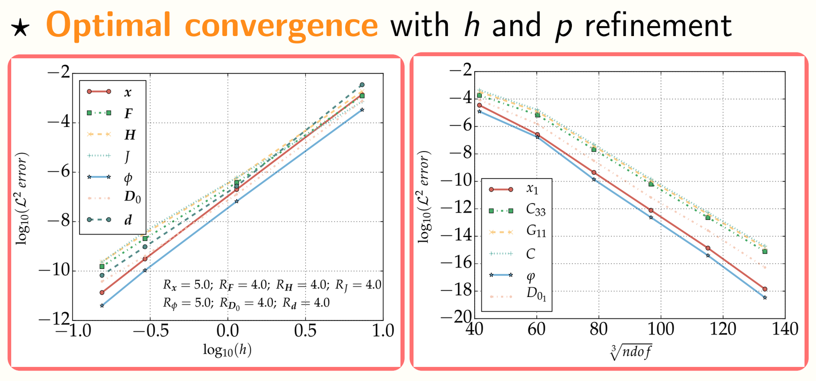
- ⋆ The solution strategy involves iteratively solving the electrostatic equations
- ⋆ This is followed by solving a consistent incrementally linearised elasticity problem

$$\begin{bmatrix} \mathbf{K}_{\boldsymbol{uu}_0}^n & \mathbf{K}_{\boldsymbol{u}\phi_0}^n \\ \mathbf{K}_{\phi\boldsymbol{u}_0}^n & \mathbf{K}_{\phi\phi_{k+1}}^{n+1} \end{bmatrix} \begin{bmatrix} \mathbf{U}_0^n \\ \mathbf{U}_{\phi_{k+1}}^{n+1} \end{bmatrix} = -\begin{bmatrix} \mathbf{R}_{\boldsymbol{u}0}^n \\ \mathbf{R}_{\phi_{k+1}}^{n+1} \end{bmatrix}$$

Per increment — Per iteration



- ⋆ Optimal convergence with h and p refinement



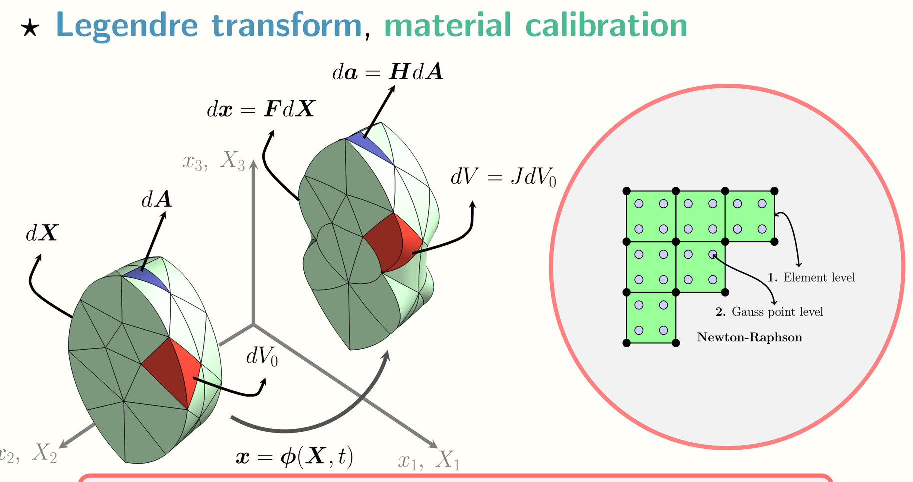
Monolithic Coupling for Large Deformation Electromechanics

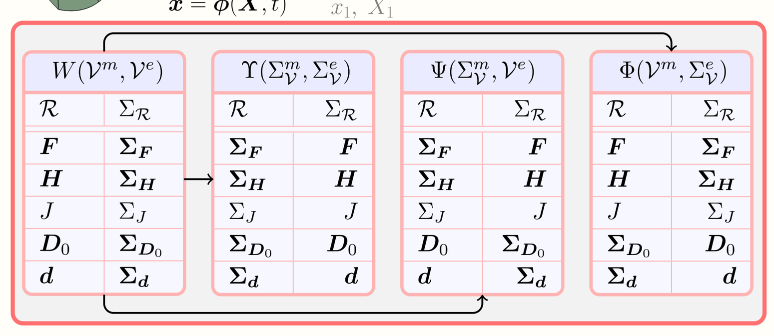
Florence (Open Source) | Python

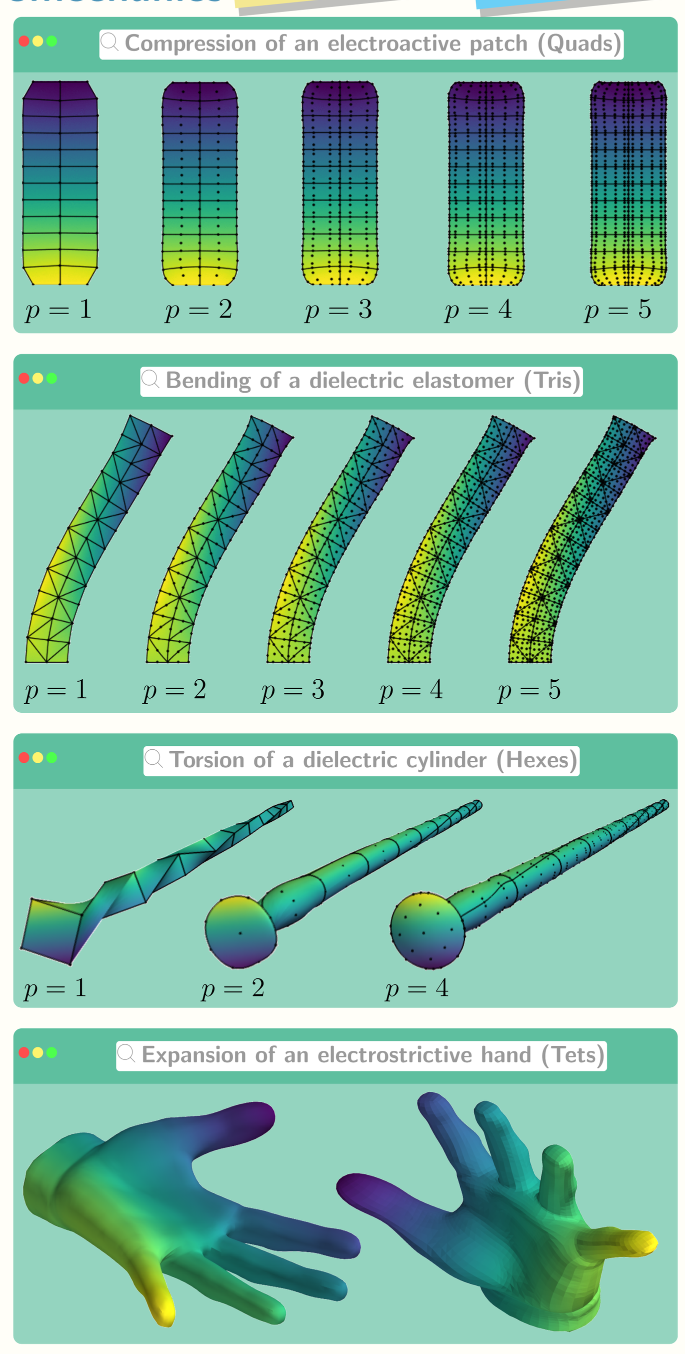
- ⋆ The set of governing equations

$$\begin{aligned}
\mathrm{DIV}\boldsymbol{P} + \boldsymbol{f}_0 &= \rho\frac{\partial^2\phi}{\partial t^2} && \text{in } V \times [0,t] \\
\boldsymbol{P}\boldsymbol{N} &= \boldsymbol{t}_0 && \text{on } \partial_t V \times [0,t] \\
\phi &= \bar{\phi} && \text{on } \partial_u V \times [0,t] \\
\phi &= \phi_0 && \text{on } \bar{V} \times 0 \\
\dot{\phi} &= \dot{\phi}_0 && \text{on } \bar{V} \times 0 \\
\mathrm{DIV}\boldsymbol{D}_0 - \rho_0^e &= 0 && \text{in } V \times [0,t] \\
\boldsymbol{D}_0 \cdot \boldsymbol{N} &= -\omega_0^e && \text{on } \partial_\omega V \times [0,t] \\
\psi &= \bar{\psi} && \text{on } \partial_\psi V \times [0,t]
\end{aligned}$$

- ⋆ Legendre transform, material calibration



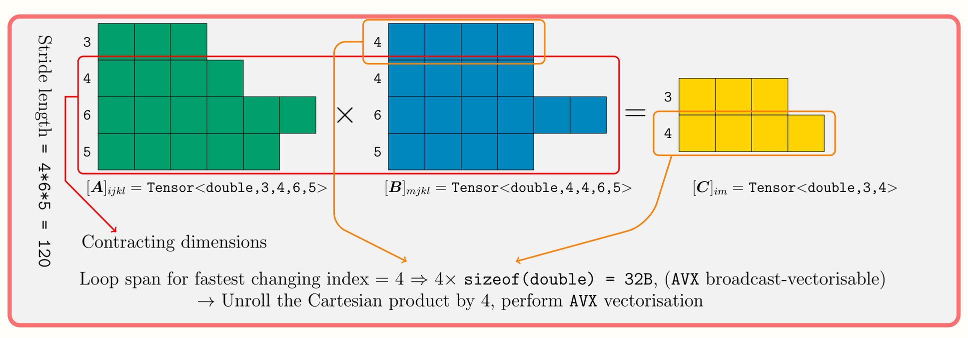


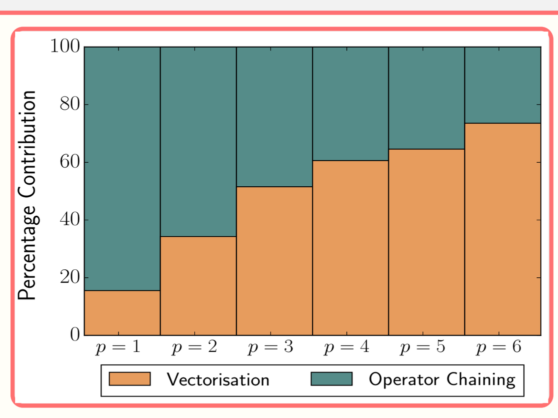


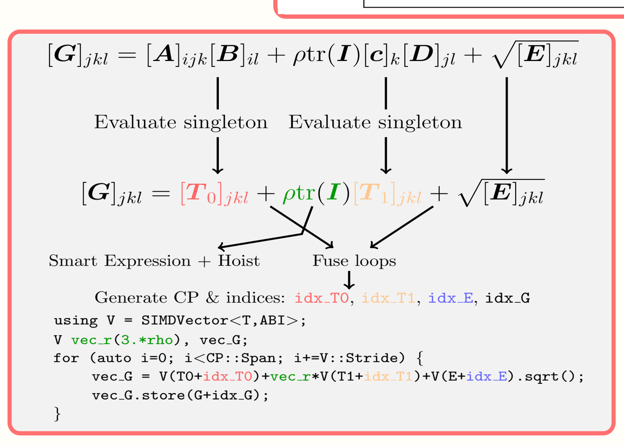
A Domain-Aware High Performance FEM Compiler

Fastor (Open Source) | C++11

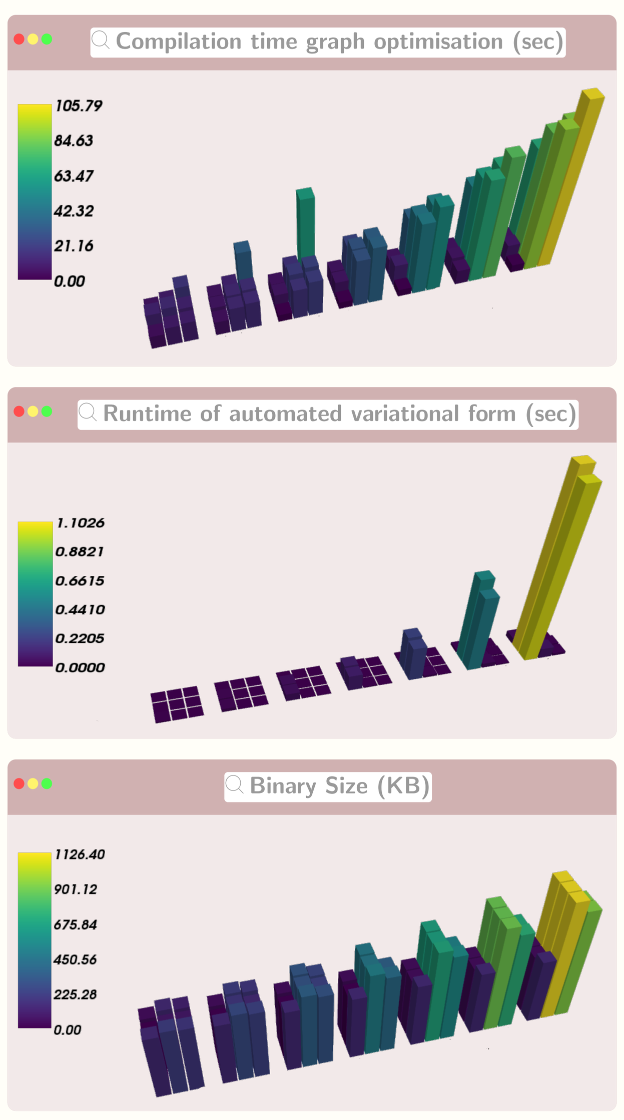
- ⋆ A SIMD based data parallel and thread parallel tensor contraction compiler is developed for variational forms
- ⋆ The framework is capable of graph search optimisation for restrusting FE expressions







Also used by other independent research groups



Speed-up over various implementation

p	Explicit SIMD	Auto Vectoriser	Classic ×
$p=3$	1.183	3.321	6.277
$p=5$	1.604	6.114	8.313

Future Outlook

4 work packages developed

On-going work:

- ⋆ Inclusion of vacuum effects through boundary element coupling
- ⋆ Studying parallel scalability in more detail
- ⋆ Re-structuring anisotropic and boundary layer mesh generation process for efficiency

Future work:

- ⋆ Extending the framework to include immersed techniques for FSI
- ⋆ Further analysis of flexoelectricity and size-dependent phenomena
- ⋆ Extending the platform to direct hex dominant mesh generation

Related Publications:

1. Roman Poya, Rogelio Ortigosa, Antonio J. Gil, A high performance high order curvilinear finite element framework for electromechanics: from small to giant deformations, *IJNME*, Submitted, (2017).
2. Roman Poya, Antonio J. Gil, Rogelio Ortigosa, A high performance data parallel tensor contraction framework: Application to coupled electro-mechanics, *Computer Physics Communications*, To Appear, (2017)
3. Roman Poya, Ruben Sevilla, Antonio J. Gil, A unified approach for a posteriori high-order curved mesh generation using solid mechanics, *Computational Mechanics*, Vol 58, Issue 3, pp 457-490, (2016)
4. Paul David Ledger, Antonio J. Gil, Roman Poya, Marcel Kruip, Ian Wilkinson, Scott Bagwell, Solution of an industrially relevant coupled magnetomechanical problem set on an axisymmetric domain, *Applied Mathematical Modelling*, Vol 40, Issue 3, pp 1959-1971, (2016)
5. Roman Poya, Antonio J. Gil, Paul D. Ledger, A computational framework for the analysis of linear piezoelectric beams using hp-FEM, *Computers and Structures*, Vol 152, pp 155-172, (2015)

Software Subpacakges:

1. PostMesh: A posteriori curvilinear mesh generator — https://github.com/romeric/PostMesh
2. Fastor: A SIMD optimised data parallel tensor contraction framework — https://github.com/romeric/Fastor
3. FEAPB: A *hp* finite element framework for piezoelectric beams — https://github.com/romeric/FEAPB
4. Florence: A comprehensive framework for coupled multi-physics problems — https://github.com/romeric/florence

Funding & Collaboration:

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